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Self-similar analytical solutions for blast waves in inhomogeneous atmospheres with frozen-in-magnetic field

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Abstract. In the present paper, we have made an attempt to study the effects of the presence of a magnetic field on the cavity formation inside a blast wave propagating into a perfectly conducting gas with density varying as some power of distance from the plane or line of explosion. In order to obtain the closed form solutions for the flow variables inside the blast wave and to solve the problem of cavity formation analytically, a relation is taken between the ordinary pressure and the total pressure. It is found that if the value of the inhomogeneity index α is greater than a critical value α_c (a function of M_A – Alfven Mach Number, γ – adiabatic index and i – wave geometry index), a contact discontinuity appears at some point inside the blast wave and the cavity formation occurs. The effect of the presence of magnetic field is found to increase the tendency of cavity formation.

PACS. 47.40.-x Compressible flows; shock and detonation phenomena – 47.65.+a Magnetohydrodynamics and electrohydrodynamics

1 Introduction

When a large amount of energy is suddenly released in a relatively small region, a disturbance headed by a strong shock wave called 'blast wave' is produced and propagates into the surrounding gaseous medium. Since the early work of Taylor [1], a considerable number of publications on the blast wave propagation have appeared in the literature, including treatises and reviews such as those of Sedov [2], Sakurai [3], Lee *et al.* [4] and Korobeinikov [5]. The pioneering studies of this phenomena (Taylor [1] and Sedov [2]) were based on self-similarity considerations and found in good agreement with experimental results. Analytical solutions for the blast wave propagation in homogeneous and non-homogeneous medium have been obtained by Rogers [6], Bach and Lee [7], Laumbach and Probstein [8], Sachdev [9] and many others. Laumbach and Probstein [8], and Sachdev [9] used an approach, based on the shock propagation theory of Brinkley and Kirkwood [10], which permits a simple analytical solution to be obtained directly from integrated form of the fundamental equations. Rogers [6] obtained the closed form solution for spherical blast wave, and discussed about the formation of cavity in the region behind the shock front in the case when the density of the undisturbed medium varies as $r^{-\alpha}$, where r is distance from the point of explosion and α a constant.

Since at high temperatures, the normal gases like hydrogen and even helium are ionized and the medium behaves like a medium of very high electrical conductivity, the electromagnetic effects may also be significant. One is thus led to study the interaction of electromagnetic field with gasdynamic forces. There are many problems in which the energy in the electric field is much smaller than that in the magnetic field. In these cases, all the electromagnetic quantities may be expressed in terms of magnetic field. As a result, one may consider only the interaction between the magnetic field and the gasdynamic field. This analysis is the well known "Magnetogasdynamics". Study of magnetogasdynamic shock waves and detonations has considerable applications in various astrophysical, geophysical and technological problems, for example, propagation of a flare produced shock in the solar wind (Lee and Chen [11], Summers [12]), generation of gas-ionizing shock waves by magnetic compression to produce high temperature plasma samples in laboratory (Sakurai [3], Nagayama [13]) and cylindrical blast wave produced by a wire explosion (Sakurai [3], Christer and Helliwell [14]).

Similarity solutions for the blast wave phenomena in magnetogasdynamics have been obtained by a number of authors, for example, Pai [15], Cole and Greifinger [16], Sakurai [3], Christer and Helliwell [14], Ray [17], Summers [12], Verma, Vishwakarma and Sharan [18], and Singh and Singh [19], but little attention is given to the formation of cavity in the region behind the shock front. In the present paper, we have made an attempt to study the effects of the presence of a magnetic field on the cavity formation inside a blast wave propagating into

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a perfectly conducting gas with density varying as $r^{-\alpha}$, r being the distance from the plane or line of explosion and α a constant. In order to obtain the closed form solution for the flow variables inside the blast wave and to solve the problem of cavity formation analytically the pressure is taken in the form (Golitsyn [20], and Ojha, Nath and Takhar [21])

$$p = \beta p^*, \quad (0 < \beta < 1) \tag{1.1}$$

where p and p^* are the gas pressure and the total pressure (sum of gas pressure and magnetic pressure), respectively. It is found that if the value of the inhomogeneity index α is greater than a critical value α_c (a function of M_A-Alfven Mach number, γ– adiabatic index and i – wave geometry index), a contact discontinuity appears at some point inside the blast wave and the cavity formation occurs. The effect of the magnetic field is found to increase the tendency of cavity formation.

2 Equations of motion and boundary conditions

We seek solutions of the magnetogasdynamic equations which govern a planar or cylindrically symmetric flow of a perfectly conducting gas across a magnetic field which is normal to the flow (axial magnetic field in the case of cylindrical symmetry). The gas is supposed ideal, and endowed with an adiabatic index γ. Viscosity, thermal conductivity and electrical resistivity are ignored.

The model equations therefore consists of the usual statement of conservation of mass, momentum and energy, and a magnetic field equation. The magnetic field equation contains all the relevant information needed from Maxwell's equations and Ohm's law; the diffusion term is omitted from it by virtue of the assumed perfect electrical conductivity. The omission of the diffusion term from the magnetic field equation means that the magnetic field under consideration is such that the magnetic lines of forces are "frozen" into the material (Cowling [22]). These equations govern the mass density ρ, the velocity u, the pressure p, and the magnetic field h, all of which depend only on the time t and the distance r from the initial plane or axis of disturbance. They are (Verma and Vishwakarma [23], Whitham [24], Sakurai [3])

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{i\rho u}{r} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu h}{\rho}\frac{\partial h}{\partial r} = 0, \tag{2.2}$$

$$\frac{\partial}{\partial t}(p\rho^{-\gamma}) + u\frac{\partial}{\partial r}(p\rho^{-\gamma}) = 0, \tag{2.3}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{ihu}{r} = 0, \tag{2.4}$$

where μ is the magnetic permeability, and $i = 0$, 1 for planar and cylindrical symmetries, respectively.

Taking the pressure in the form as given in equation (1.1), and using the relation

$$\frac{h}{\rho} = \text{const.}, \tag{2.5}$$

as obtained from equations (2.1) and (2.4), the effective speed of sound c is defined by

$$c^2 = \frac{dp^*}{d\rho} = a^2 + b^2 = \frac{\gamma^* p^*}{\rho}, \tag{2.6}$$

where

$p^* = p + \frac{\mu h^2}{2}$ is the total pressure,

$a^2 = \frac{\gamma p}{\rho}$ is the square of the sound speed,

$b^2 = \frac{\mu h^2}{\rho}$ is the square of the Alfven speed,

and

$$\gamma^* = \gamma\beta + 2(1-\beta). \tag{2.7}$$

Assuming the flow to be isentropic and β to be a constant (Verma and Vishwakarma [25], Ojha, Nath and Takhar [21]), equation (2.6) gives rise to the energy equation in the form

$$\frac{\partial}{\partial t}(p^*\rho^{-\gamma^*}) + u\frac{\partial}{\partial r}(p^*\rho^{-\gamma^*}) = 0, \tag{2.8}$$

which is the same as if γ and p were replaced by γ^* and p^* in the equation (2.3).

Equation of energy in the form (2.8) is true only when equation (2.5) holds and β has a constant value. The assumption that β is constant, physically means that the gas pressure and the magnetic pressure are in a constant ratio, that is, the speed of sound and the Alfven speed are in a constant ratio. Also, equation (2.5) holds only when the electrical conductivity of the gas is infinite and the magnetic field is axial in the case of cylindrically symmetric flow and normal to the flow in the case of planar symmetry. As already stated, we have considered all these assumptions, in order to obtain closed form similarity solutions of the problem of cavity formation inside a magnetogasdynamic blast wave.

Using (2.1), equation (2.8) becomes

$$\frac{\partial p^*}{\partial t} + u\frac{\partial p^*}{\partial r} + \gamma^* p^*\left(\frac{\partial u}{\partial r} + \frac{iu}{r}\right) = 0. \tag{2.9}$$

From equations (2.1, 2.2) and (2.9), we get

$$\frac{\partial E}{\partial t} + \frac{1}{r^i}\frac{\partial}{\partial r}\left(r^i\, u\, I\right) = 0, \tag{2.10}$$

where

$$E = \frac{\rho u^2}{2} + \frac{p^*}{\gamma^* - 1} \text{ and } I = E + p^*.$$

It is assumed that, at time $t = 0$, an explosion takes place over a plane or along a line, accompanied by release of a finite amount of energy E_0. A plane or cylindrical strong shock is instantaneously formed which begins to propagate outward into a perfectly conducting gas at rest. The density and the magnetic field ahead of the shock are assumed to vary as

$$\rho_0 = \rho_c R^{-\alpha}, \tag{2.11}$$

and

$$h_0 = h_c R^{-\alpha_1}, \tag{2.12}$$

where R is the distance of the shock surface from the plane or line of explosion, and ρ_c, h_c, α and α_1 are constants. The mass in the undisturbed state of the gas within a distance (or radius) r must be positive which requires that $\alpha < i + 1$.

The Rankine-Hugoniot boundary conditions in the case of a very strong shock take the form (Whitham [24], Summers [12])

$$u_1 = \frac{2V}{\gamma+1}, \tag{2.13}$$

$$\rho_1 = \rho_0 \frac{\gamma+1}{\gamma-1}, \tag{2.14}$$

$$p_1 = \frac{2\rho_0 V^2}{\gamma+1}, \tag{2.15}$$

$$h_1 = h_0 \frac{\gamma+1}{\gamma-1}, \tag{2.16}$$

where u_1, ρ_1, p_1, h_1 are the values of variables immediately behind the shock which is travelling with velocity $V = \frac{\mathrm{d}R}{\mathrm{d}t}$.

The flow in the disturbed region behind the shock (the blast wave region) is governed by equations (2.1, 2.2, 2.4) and (2.8) or (2.9).

3 Similarity considerations

To obtain similar solutions, we write the unknown variables in the following form

$$u = V f(x), \tag{3.1}$$
$$\rho = \rho_0 G(x), \tag{3.2}$$
$$P = \rho_0 V^2 P(x), \tag{3.3}$$
$$\sqrt{\mu} h = \sqrt{\rho_0} V H(x), \tag{3.4}$$

where f, G, P and H are the functions of the non-dimensional variable $x = r/R(t)$ only. The shock front is represented by $x = 1$.

If the initial energy is assumed to be negligible, the energy release E_0 equals the total energy within the blast wave region, *i.e.*,

$$E_0 = \sigma_i \int_0^R \left(\frac{1}{2}\rho u^2 + \frac{p}{\gamma-1} + \frac{\mu h^2}{2}\right) r^i \mathrm{d}r, \tag{3.5}$$

where $\sigma_i = 2i\pi + (1 - i)$.

Applying the similarity transformations (3,1) to (3.4) in the relation (3,5), we find that the motion of the shock front is given by the equation

$$V = \frac{\mathrm{d}R}{\mathrm{d}t} = \left(\frac{E_0}{\sigma_i \rho_c J}\right)^{1/2} R^{(\alpha-i-1)/2}, \tag{3.6}$$

where $J = \int_0^1 \left(\frac{1}{2}Gf^2 + \frac{P}{\gamma-1} + \frac{H^2}{2}\right) x^i \mathrm{d}x$.
Equation (3.6), on integration, yields

$$R = \left(\frac{i-\alpha+3}{2}\right)^{2/(i-\alpha+3)} \left(\frac{E_0}{\sigma_i \rho_c J}\right)^{1/(i-\alpha+3)} t^{2/(i-\alpha+3)} \tag{3.7}$$

and therefore,

$$V = \frac{2}{i-\alpha+3} \frac{R}{t}. \tag{3.8}$$

After using the similar transformations, the equations (2.1, 2.2, 2.4, 2.9) and (2.10) change into the following set of ordinary differential equations:

$$(x-f)\frac{G'}{G} = f' + \frac{if}{x} - \alpha, \tag{3.9}$$

$$(x-f)f' = \frac{P^{*\prime}}{G} + \left(\frac{\alpha-1-i}{2}\right) f, \tag{3.10}$$

$$(x-f)\frac{H'}{H} = f' + \frac{if}{x} - \left(\frac{i+1}{2}\right), \tag{3.11}$$

$$(x-f)P^{*\prime} = \gamma^* P^* + \left(f' + \frac{if}{x}\right) - (i+1)P^*, \tag{3.12}$$

$$(x^i f I)' = x^{i+1} E' + (i+1) E x^i, \tag{3.13}$$

where primes denote derivatives with respect to x, and

$$P^* = P + \frac{1}{2}H^2.$$

The boundary conditions (2.13) to (2.16) change into

$$f(1) = \frac{2}{\gamma+1}, \tag{3.14}$$

$$G(1) = \frac{\gamma+1}{\gamma-1}, \tag{3.15}$$

$$P(1) = \frac{2}{\gamma+1}, \tag{3.16}$$

$$H(1) = \frac{\gamma+1}{\gamma-1} M_A^{-1}, \tag{3.17}$$

where $M_A = V / \left(\frac{\mu h_0^2}{\rho_0}\right)^{1/2}$ is the Alfven Mach number, and $2\alpha_1 = i + 1$, as obtained from similarity considerations.

At the shock, the relation (1.1) enables us to obtain the value of β in terms of γ and M_A as

$$\beta = \frac{2/(\gamma+1)}{\frac{2}{(\gamma+1)} + \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2 M_A^{-2}}.$$

4 Analytical solutions

Integrating the equation (3.13), we get

$$fI = xE \tag{4.1}$$

which can be written in the form

$$\frac{P^*}{G} = \frac{(x-f)(\gamma^*-1)f^2}{2(\gamma^* f - x)}\cdot \tag{4.2}$$

By taking certain linear combination of equations (3.9) and (3.12), it is also possible to obtain a second integral (Rogers [6]),

$$G^{(i+1)(\gamma^*-1)/(i-\alpha+1)} = C_1 P^* x^i (x-f)^{(i+1-\gamma^*\alpha)/(i-\alpha+1)}, \tag{4.3}$$

where C_1 is the constant of integration and its value can be easily calculated from the boundary conditions. From equations (4.2) and (4.3), we get

$$G = C_2 \left\{ \frac{f^2(x-f)}{(\gamma^* f - x)} \right\}^{\frac{(i+1-\alpha)}{(\gamma^*-2)(i+1)+\alpha}} \times \left\{ x^i (x-f) \right\}^{\frac{(i+1-\gamma^*\alpha)}{(\gamma^*-2)(i+1)+\alpha}}, \tag{4.4}$$

where C_2 is suitable constant. Using (4.4) in (4.2), we get

$$\begin{aligned} P^* &= P + \frac{1}{2}H^2 \\ &= C_3 \left\{ \frac{f^2}{\gamma^* f - x} \right\}^{\frac{(i+1)(\gamma^*-1)}{(i+1)(\gamma^*-2)+\alpha}} x^{\frac{i(i+1-\gamma^*\alpha)}{(i+1)(\gamma^*-2)+\alpha}} \\ &\quad \times (x-f)^{\frac{\gamma^*(i+1-\alpha)}{(i+1)(\gamma^*-2)+\alpha}}, \end{aligned} \tag{4.5}$$

where C_3 is a constant, which can be determined with help of the boundary conditions. Further, from (1.1), we have

$$H = \{2(1-\beta)P^*\}^{1/2}. \tag{4.6}$$

The function $f(x)$ is yet to be determined. Using (3.12) and (4.2) in (3.10), we get a differential equation in f and x,

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{f}{x} \left\{ \frac{Af^2 + Bfx + Cx^2}{A_1 f^2 + B_1 fx + 2x^2} \right\}, \tag{4.7}$$

where

$$A = -\gamma^*(\gamma^*-1)i,\ B = -(\gamma^*\alpha - 2\gamma^* i - 2\gamma^* + i + 1),$$
$$C = (\alpha - i - 1),\ A_1 = \gamma^*(\gamma^*+1),\ B_1 = -2(\gamma^*+1).$$

Equation (4.7) shows that no singularity can occur in the velocity profile except perhaps on $x = 0$. Since $f'(x)$ is a function of f/x, any line $f = kx$ will satisfy the differential equation (4.7), provided that

$$(A_1 - A)k^3 + (B_1 - B)k^2 + (2 - C)k = 0.$$

The roots of this cubic equation in k are

$$k_1 = 0, \quad k_2 = (1/\gamma^*), \quad k_3 = (i+3-\alpha)/(\gamma^* i + \gamma^* + 1 - i).$$

The importance of these lines ($f/x = k_1, k_2, k_3$) lies in the fact that no solution curve of (4.7) can cross them at any point except at the origin. They, therefore, act as guide lines for the velocity profiles. The particular solution, which we require, is that which satisfies the boundary condition $f(1) = \frac{2}{\gamma+1}$. This gives rise to three types of solutions according to

$$\frac{2}{\gamma+1} \lesseqgtr \frac{i+3-\alpha}{\gamma^* i + \gamma^* + 1 - i},$$

which can be expressed in terms of α as $\alpha \lesseqgtr \alpha_c$, where

$$\alpha_c = (i+1) - \frac{2}{\gamma+1}\{(\gamma^* - \gamma) + i(\gamma^* - 1)\}\cdot$$

Thus there exists a critical value α_c of α such that the velocity profiles for $\alpha < \alpha_c$ will be between the lines

$$f = \frac{1}{\gamma^*}x \quad \text{and} \quad f = \frac{(i+3-\alpha)x}{(\gamma^* i + \gamma^* + 1 - i)}\cdot$$

The velocity profiles for $\alpha > \alpha_c$ will lie above these lines, and for $\alpha = \alpha_c$ will coincide with the line $f = \frac{2}{\gamma+1}x$.

The self-similar solution of the present problem in the case of $\alpha = \alpha_c$, is obtained in a very simple form, from (4.4, 4.5) and (4.6), as

$$\begin{aligned} f &= \frac{2}{\gamma+1}x, \\ G &= \frac{\gamma+1}{\gamma-1}x^n, \\ P^* &= \left\{ \frac{2}{\gamma+1} + \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2 M_A^{-2} \right\} x^{n+2}, \\ H &= \frac{\gamma+1}{\gamma-1} M_A^{-1} x^{(n+2)/2}, \end{aligned} \tag{4.8}$$

where

$$n = \frac{(i+1)(3+i) - \alpha_c\,[2 + \gamma^*(i+1)]}{(i+1)(\gamma^*-2) + \alpha_c}\cdot$$

The solution of the homogeneous differential equation (4.7) is

$$\left(\frac{\gamma+1}{2}\frac{f}{x}\right)^{a_1} \left\{ \frac{a_6^2 - a_7^2}{a_6^2 - a_8^2} \right\}^{\frac{a_1 a_2}{4a_3}} \times \left[\left\{ \frac{a_6 - a_7}{a_6 + a_7} \right\} \left\{ \frac{a_6 + a_8}{a_6 - a_8} \right\} \right]^{\frac{a_1(4a_3a_5 - a_2a_4)}{8a_3^2 a_6}} = \frac{1}{x}, \tag{4.9}$$

where,

$$a_1 = \frac{2}{3+i-\alpha}, a_2 = \gamma^*(\gamma^*+1)(1-\alpha) + \gamma^*(3-\gamma^*)i,$$
$$a_3 = \gamma^*(\gamma^*-1)i + (\gamma^*+1)\gamma^*,$$
$$a_4 = \gamma^*\alpha - (2\gamma^*-1)i - 4\gamma^* - 1,$$
$$a_5 = \gamma^* + (\alpha - 2) - (2-\gamma^*)i, \quad a_6 = \frac{\sqrt{a_4^2 - (8a_3/a_1)}}{2a_3},$$
$$a_7 = \frac{f}{x} + \frac{a_4}{2a_3}, \quad a_8 = \frac{2}{\gamma+1} + \frac{a_4}{2a_3}\cdot$$

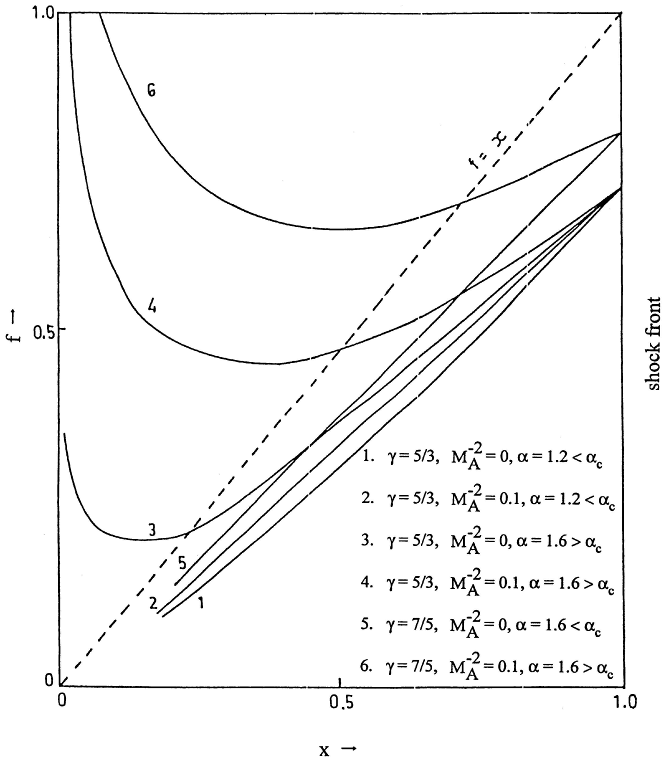

Fig. 1. Variation of velocity with distance in the region behind cylindrical shock wave.

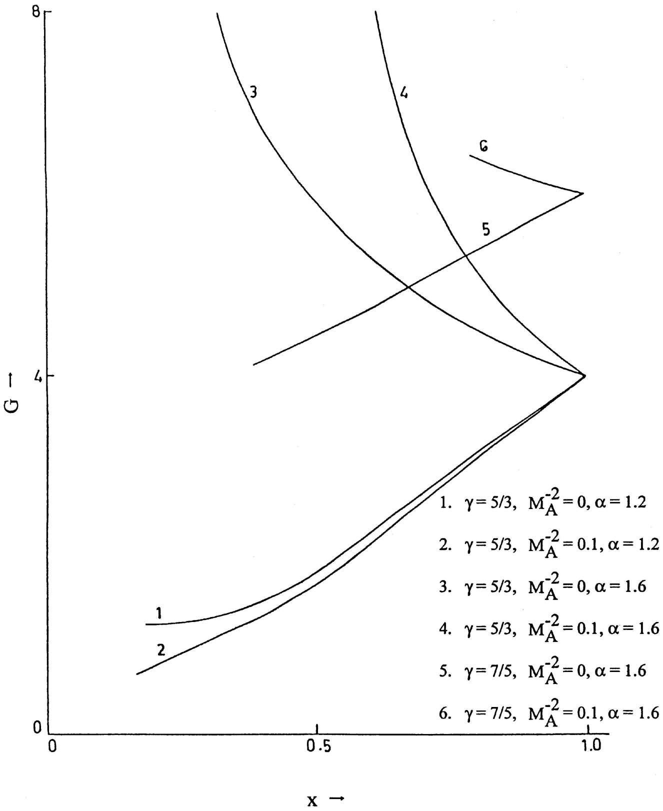

Fig. 2. Variation of density with distance in the region behind cylindrical shock wave.

The equations (4.4, 4.5, 4.6) and (4.9) form the complete set of self-similar analytical solutions for the blast wave problem under the present investigations. These solutions are analogous to the solutions obtained by Rogers [6] in ordinary gas dynamics.

5 Results and discussion

Calculations have been performed from equations (4.4, 4.5, 4.6) and (4.9) to obtain the dimensionless flow variables f, G, P and $\frac{1}{2}H^2$ in the region behind the shock front for $\gamma = \frac{7}{5}, \frac{5}{3}$; $\alpha = 1.2, 1.6$; $M_A^{-2} = 0, 0.1$; and $i = 1$. The value $M_A^{-2} = 0$ corresponds to the non-magnetic case. $M_A^{-2} = 0.001$ corresponds to a very weak ambient magnetic field, and Rosenau and Frankenthal [26] have shown that the corresponding flow is very close to that of the non-magnetic case. They have also found that when $M_A^{-2} = 0.01$, the magnetically dominated layer in the flow-field behind the shock is thicker than that for $M_A^{-2} = 0.001$, and when $M_A^{-2} = 0.1$ the influence of magnetic forces is evident throughout the flow-field. We, therefore, have taken $M_A^{-2} = 0.1$ to study the phenomenon of cavity formation inside the blast wave in presence of magnetic field.

The variation of the flow variables, are shown in Figures 1 to 4. It is found that, for $\alpha > \alpha_c$, the density G increases (see Fig. 2), the gas pressure P and the magnetic pressure $\frac{1}{2}H^2$ decrease sharply as we move inward from the shock front (see Figs. 3 and 4), while the velocity f decreases up to a critical point which is the point of intersection of the velocity profile with the line $f = x$ (see Fig. 1). At this critical point ($x = x_0$), the density G becomes exceedingly large, the gas pressure P and the magnetic pressure $\frac{1}{2}H^2$ become small, whereas the velocity f suddenly begins to increase. This shows that a discontinuity surface $r = S(t)$ arises at the point $x = x_0$, where

$$u = Vf(x_0) = Vx_0, \tag{5.1}$$

and

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(Rx_0) = Vx_0. \tag{5.2}$$

Thus the discontinuity surface S(t) is a contact surface, which disallows mass flux across it but moves as a solid with the fluid velocity (since $u = \frac{\mathrm{d}S}{\mathrm{d}t}$ from Eqs. (5.1) and (5.2)).This results in the cavity formation. A physical explanation of cavity formation may be given as follows:

Very high increase in the density and high decrease in the gas pressure and the magnetic pressure at the critical point inside a blast wave results in conversion of huge amount of energy (internal and magnetic) into kinetic energy. The concentrated matter, therefore, moves suddenly with a very high speed causing the formation of cavity.

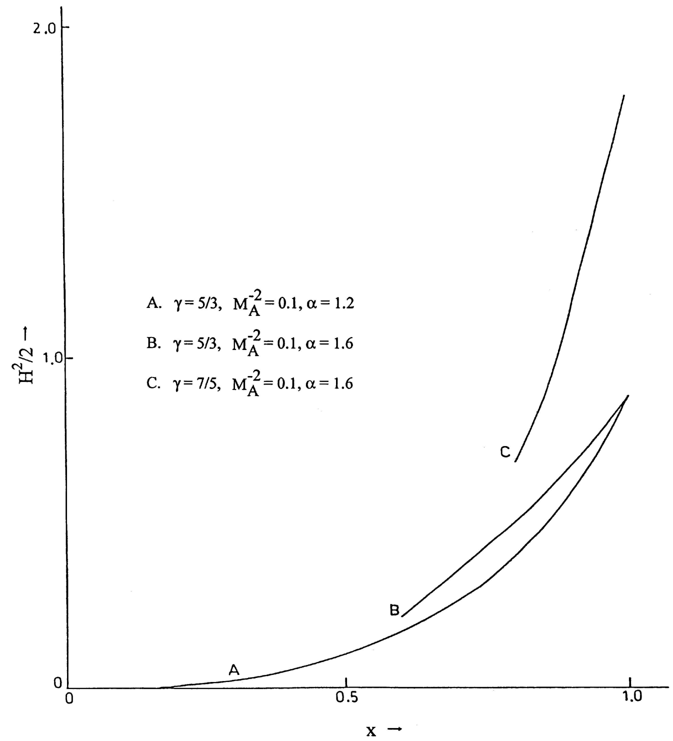

Fig. 3. Variation of magnetic pressure with distance in the region behind cylindrical shock wave.

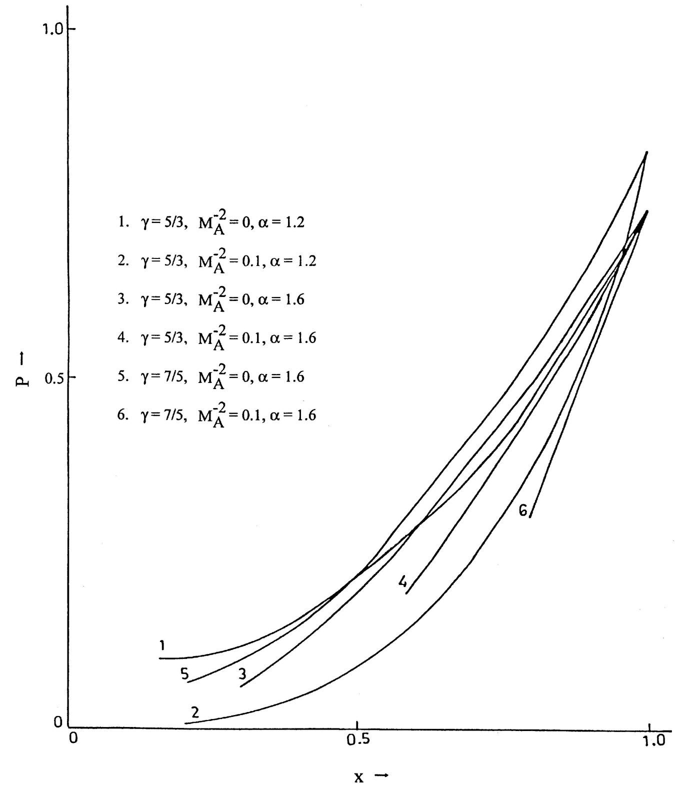

Fig. 4. Variation of pressure with distance in the region behind cylindrical shock wave.

From Figure 1 and Table 1, we find the main effects of increasing the ambient magnetic field as follows:

(a) Curves (3) and (4) in Figure 1 show that the point of intersection of the line $f = x$ with the velocity profile is nearer to the shock front for $M_A^{-2} = 0.1$ in comparison with that for $M_A^{-2} = 0$. Therefore an increase in the ambient magnetic field reduces the distance between the inner contact surface and the shock front.

(b) In Figure l, curve (5) is the velocity profile for $M_A^{-2} = 0.1$ and curve (6) in that for $M_A^{-2} = 0.1$ with the same values of α and γ. The line $f = x$ intersects the curve (6) but not the curve (5). This shows that an increase in the ambient magnetic field increases the tendency of the cavity formation inside the blast wave region.

(c) Table 1 shows that α_c decreases by an increase in M_A^{-2}, which ultimately concludes the same result as in (b) above.

Thus the presence of magnetic field, in general, increases the tendency of automatic occurrence of contact discontinuity and the formation of cavity inside the blast wave region. This is perhaps due to the fact that, in presence of magnetic field, there is conversion of not only internal energy but also of the magnetic energy into the kinetic energy at the critical point inside the blast wave region, which is responsible for the cavity formation.

For $\alpha < \alpha_c$, the solution curves are approaching the origin and the velocity profile does not intersect with the line $f = x$. Therefore, in general, there is non-occurrence of discontinuity surface and not formation of cavity inside the blast wave region.

Table 1. Variation of α_c with γ and M_A^{-2}, for $i = 1$.

γ	$\frac{7}{5}$	$\frac{7}{5}$	$\frac{7}{5}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$
M_A^{-2}	0	0.01	0.1	0	0.01	0.1
α_c	1.6667	1.4890	0.9831	1.5	1.4518	1.2419

One can see from figures that cavitation does or does not occur as a function of α, for given values of M_A and γ. For example, in Figure 1, curves 2 and 4 are the velocity profiles for the same values of M_A and γ but for different values of α. Curve 2 corresponds to $\alpha = 1.2$ which is less than $\alpha_c(= 1.2419)$ in this case, whereas curve 4 corresponds to $\alpha = 1.6$ which is greater than α_c. Curve 4 intersects with the line $f = x$ at a critical point where a discontinuity (contact) surface occurs which moves with the fluid velocity causing the formation of a cavity. Curve 2 does not intersect with the line $f = x$ and no cavity is formed.

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